Random Convection under Conditions of Weightlessness

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The nature of the transport process between a fluid and its enclosing surface is considered in the presence of random disturbances and, in particular, for conditions likely to prevail in space devices. The argument is developed that disturbances normally present in the motion of such devices may result in relatively effective transport mechanisms. On the basis of assumptions regarding the nature of the disturbances and their mode of occurrence, a number of circumstances are analyzed. The resulting transport rates generally are much greater than would be calculated for the process that would be expected in the absence of all disturbances.

Nomenclature

c_p = specific heat
F = $\alpha\tau_c/s^2$, the disturbance Fourier number
Fo = $\alpha\tau/x^2$ or $\alpha\tau/r^2$, Fourier number
h = surface (or convection) coefficient
h_{fg} = latent heat of vaporization
k = thermal conductivity
m'' = mass rate of condensation/unit area
n = positive integer in the probability distribution
Nu = hs/k, Nusselt number, where k is the thermal conductivity of the fluid
q'' = rate of heat transfer/unit area
Q = amount of heat transferred
Q'' = amount of heat transferred/unit area
s = significant dimension
t = temperature
y = F/F_m
Y = thickness of a condensate film
α = thermal diffusivity
θ = temperature excess $(t - t_\infty)$
ρ = fluid density
τ = time

Subscripts

c = based upon the time interval between disturbances
m = mean or most probable value
0 = at the fluid-solid interface

Introduction

Many space devices contain enclosures filled with a gas, vapor, or liquid (or a mixture of phases), e.g., condenser-radiators, heat exchangers, vapor generators, and conditioned spaces. In the presence of gravity, the inevitable density differences that arise in mass or thermal transfer processes result in relative fluid motion that enhances transfer.

However, in free motion in space, the body force is zero. Therefore, natural convection effects are nominally absent. Under this condition the proper theory for the various types of transport processes, in the absence of forced flow, is the appropriate conduction theory. This type of analysis predicts very low transfer rates in many important processes, e.g., in boiling or condensation where the vapor or liquid blankets the surface, or in cooling a long cylinder immersed in a fluid of relatively large extent from which no heat could be transferred in steady state with a finite temperature difference. Such results are obtained from conduction analysis for many circumstances and are very unfortunate because of the rigid criteria necessary in the design of devices for space. The purpose of this paper is to suggest practical considerations that imply "random" convection processes in space devices and to show the effect that such processes would have upon transport rates.

Space devices of any size and complexity have associated with their operation many processes and events that supply appreciable impulses to the device while in free motion. Such impulses arise from internal mechanical events, particle impacts, attitude control measures, motion of occupants, etc. Even when subject to the relatively small acceleration of an electric propulsion system, these effects, as well as those due to uneven thrust from the propulsion system, may be significant. The net effect of such disturbances on the motion of a device whose attitude is controlled will be successive small fluctuations about an instantaneous mean velocity and orientation in space.

Consider, for example, a single phase fluid in an enclosure subject to these disturbances. The effect of the fluctuations will be transferred to the fluid by normal and shear stresses. The fluctuations in velocity principally cause normal stresses that will result in little relative motion between the fluid and the enclosure walls if the fluctuations are relatively slow compared to the time necessary for the propagation of pressure disturbances. However, the fluid is affected by changing orientation primarily by shear stresses, which are relatively small and act only during the period of relative motion. Therefore, the fluid maintains essentially the same orientation in space, whereas the enclosure fluctuates in orientation. The result of these considerations is that it is perhaps reasonable to assume that there is an important design application in which there is relative motion between bounding surfaces and enclosed fluids and that these motions may be idealized as a sequence of abrupt relative displacements spaced by time intervals τ_c. Various objects in such an enclosure, but attached to it, also would experience a displacement with respect to the fluid with which they are in contact.

The foregoing idealizations provide a basis for theories of transport processes in various circumstances. Assumptions must be made concerning the time interval τ_c because, to the present writer's knowledge, no measurements have been made in space concerning either the magnitude of these disturbances or their spacing in time. The magnitude of the relative motion is not important as long as it is at least the same order of size as the transfer element of interest. However, τ_c has an important effect.

Considering the more complex among various proposed space devices, it is perhaps reasonable to postulate that there will be many unconnected causes of such disturbances and that, therefore, τ_c is a randomly distributed variable with a probability distribution $f(\tau_c)$. The form of $f(\tau_c)$ and the most probable value of the time interval τ_m will depend upon the nature of the effects that produce the disturbances and upon

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their number. For the kinds of transport processes analyzed to date, it appears that τ_m is the principal variable; various reasonable choices of $f(\tau_c)$ produce similar results.

Analysis

In order to assess the effect of disturbances upon transport processes, a number of cases have been analyzed. The first to be discussed is heat transfer to a single-phase fluid. The relative motion, between the bounding solid and the fluid, is assumed to amount to an instantaneous relative displacement that is followed after time interval τ_c by another displacement. The transfer process may be approximated as pure conduction in the fluid and is a transient of duration τ_c beginning with a fluid uniformly at t_∞ with the bounding surface instantaneously raised to t_0.† This is a reasonable model for cases in which the time interval to achieve the displacement is small compared to τ_c and in which the relative displacement is at least as large as the heat transfer surface. These conditions minimize the convection effect associated with relative motion between a fluid and a bounding surface.

If the amount of heat transferred per unit of surface area in time τ_c is Q'', the average rate of heat transfer will be

$$\overline{q_0''} = \int_0^\infty \frac{Q''}{\tau_c} f(\tau_c) d\tau_c \tag{1}$$

This is written in terms of a "disturbance" Fourier number, $F = \alpha\tau_c/s^2$, where s is some significant dimension and $f(F)$ is the probability distribution of F:

$$\overline{q_0''} = \frac{\alpha}{s^2}\int_0^\infty \frac{Q''f(F)}{F}\, dF \tag{2}$$

The Nusselt number, based upon the average surface coefficient, is

$$\overline{Nu} = \bar{h}\,\frac{s}{k} = \frac{\overline{q_0''}}{(t_0 - t_\infty)}\,\frac{s}{k} = \frac{\alpha}{sk\theta_0}\int_0^\infty \frac{Q''f(F)}{F}\, dF$$

or

$$\overline{Nu} = \frac{\alpha}{sk\theta_0 F_m}\int_0^\infty \frac{Q''f(y)}{y}\, dy \tag{3}$$

where $y = F/F_m$, and F_m is the mean or most probable value of F as indicated in the subsequent discussion of the various probability distributions. The probability distribution is $f(y)$, and Q'' is to be expressed in terms of y.

A number of probability distributions appear reasonable. The exponential distribution might be used.

$$f(F) = Ce^{-CF} = (1/F_m)e^{-F/F_m} \tag{4a}$$

or

$$f(y) = e^{-y} \tag{4b}$$

where $y = F/F_m$, and F_m is the mean value of F, that is, $\bar{F}$.

Since the function $f(y)$ in Eq. (3) is to be the resultant probability distribution of a number of more or less independent effects, it is perhaps more reasonable to use the following form of the gamma distribution:

$$f_n(F/F_m) = k_n(F/F_m)^n e^{-n(F/F_m)} \tag{5}$$

where n is a positive integer and F_m is chosen as the most probable value in this case. This is the probability distribution of the sum of $n + 1$ independent variables, each exponentially distributed. This distribution approaches the Gaussian as n increases, and as $n \to \infty$ the probability goes to 1.0 at $y = 1$.‡ That is, for $n \to \infty$, all time intervals between disturbances are the same and $\tau_c = \tau_m$. In general, the most probable and mean values of F are related as follows:

$$F_m = [n/(n + 1)]\bar{F} \tag{6}$$

The constant in Eq. (5) is determined by applying the condition that the total probability is 1.0. The equation, written in terms of $y = F/F_m$, is therefore

$$f(y) = (n^{n+1}/n!)y^n e^{-ny} \tag{7}$$

The subsequent analysis shows that the effect of n on the predicted heat transfer parameters is small.

The first cases considered are the one-dimensional geometries: the isothermal flat surface, long cylinder, and sphere in extensive bodies of single-phase fluid. Coordinate systems are shown in Fig. 1. The process in the fluid after a disturbance is a pure conduction transient. The equation and boundary conditions for the plane conduction region are

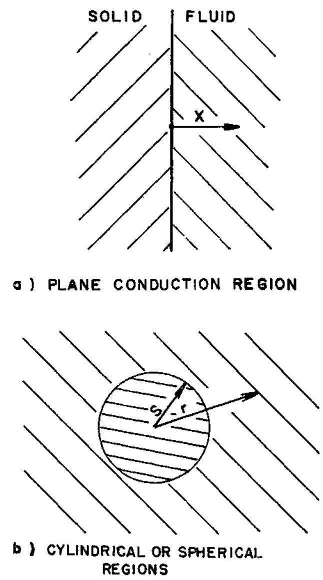

Fig. 1 Conduction regions

$$\partial^2\theta/\partial x^2 = (1/\alpha)(\partial\theta/\partial\tau) \tag{8}$$

for $\tau = 0$, $\theta = 0$ and for $\tau > 0$ at $x = 0$, $\theta = \theta_0$. The solution is (see, e.g., Carslaw and Jaeger[2])

$$\frac{\theta}{\theta_0} = \operatorname{erfc}\frac{x}{2(\alpha\tau)^{1/2}} = 1 - \frac{2}{\pi^{1/2}}\int_0^{x/2(\alpha\tau)^{1/2}} e^{-z^2}\, dz \tag{9}$$

The instantaneous flux at $x = 0$ is

$$q_0'' = -k(\partial\theta/\partial x)_0 = k\theta_0/(\pi\alpha\tau)^{1/2} = k\theta_0/s(\pi F)^{1/2}$$

The value of Q'', from $\tau = 0$ to $\tau = \tau_c$, is found:

$$Q'' = \int_0^{\tau_c} q_0''\, d\tau = \frac{k\theta_0}{(\pi\alpha)^{1/2}}\int_0^{\tau_c}\frac{d\tau}{\tau^{1/2}} = \frac{2sk\theta_0}{\alpha(\pi)^{1/2}}F^{1/2} = \frac{2sk\theta_0}{\alpha}\left(\frac{F_m}{\pi}\right)^{1/2} y^{1/2} \tag{10}$$

where the F_m introduced into Eq. (10) may be either the mean or the most probable value of F.

The statement for the spherical conduction region is

$$(\partial^2\theta/\partial r^2) + (2/r)(\partial\theta/\partial r) = (1/\alpha)(\partial\theta/\partial\tau)$$

for $\tau = 0$, $\theta = 0$ and for $\tau > 0$ at $r = s$, $\theta = \theta_0$. This problem may be reduced to that for a plane conduction region by the following change of variables: $\phi = r\theta$ and $r' = r - s$. The result is

$$\partial^2\phi/\partial r'^2 = (1/\alpha)(\partial\phi/\partial\tau) \tag{11}$$

† This case corresponds to a solid element of relatively large thermal capacity, and one solves for the average convection coefficient $\bar{h}$. There is an opposite case, of small element thermal capacity, in which a thermal flux is assigned at the surface, and the time average value of t_0 is found.

‡ For a discussion of various distributions see, e.g., Parzen.[1]

for $\tau = 0$, $\phi = 0$ and for $\tau > 0$ at $r' = 0$, $\phi = \phi_0 = s\theta_0$. Therefore, from Eq. (9),

$$\frac{\phi}{\phi_0} = \frac{r\theta}{s\theta_0} = \text{erfc}\,\frac{r'}{2(\alpha\tau)^{1/2}} = \text{erfc}\,\frac{r-s}{2(\alpha\tau)^{1/2}}$$

or

$$\theta = \theta_0 \frac{s}{r}\left(1 - \frac{2}{\pi^{1/2}}\int_0^{[(r-s)/2(\alpha\tau)^{1/2}]} e^{-z^2}\,dz\right) \qquad (12)$$

The instantaneous flux at $r = s$ and the value of Q'' are

$$q_0'' = -k\left(\frac{\partial\theta}{\partial r}\right)_s = \frac{k\theta_0}{s}\left(1 + \frac{1}{\pi^{1/2}}\frac{1}{F^{1/2}}\right)$$

$$Q'' = \frac{2sk\theta_0}{\alpha}\left[\frac{yF_m}{2} + \left(\frac{F_m}{\pi}\right)^{1/2} y^{1/2}\right] \qquad (13)$$

This is seen to be the plane conduction region solution, Eq. (10), plus a correction term $(yF_m/2)$ for curvature.

The statement of the transient problem in a cylindrical region is

$$(\partial^2\theta/\partial r^2) + (1/r)(\partial\theta/\partial r) = (1/\alpha)(\partial\theta/\partial\tau)$$

for $\tau = 0$, $\theta = 0$ and for $\tau > 0$ at $r = s$, $\theta = \theta_0$. The solution of this problem may be obtained in the form of an integral involving Bessel functions. A number of numerical estimates of the solution have been made. Of interest here are those concerning heat transfer rates at $r = s$. Jaeger[3] presented series for q_0'' in terms of F for small and for large values of F. Jaeger and Clarke[4] tabulated values of q_0''. Goldenberg[5] integrated the series of Jaeger and presented a plot of Q'' vs F. Jakob[6] tabulated a numerical evaluation of q_0'' and Q'' by Perry and Berggren.

None of these results are in a convenient form for the present purposes since the Q'' function must be integrated in Eq. (3). The data tabulated by Jakob (for $F \leqq 3.5$) may be fitted approximately by an equation of the following form:

$$Q'' = \frac{sk\theta_0}{\alpha}\left[c(F)^b + 2\left(\frac{F}{\pi}\right)^{1/2}\right] \qquad (14)$$

The values of b and c is 0.95 and 0.4, respectively. However, as is shown later, the present analysis is of interest mainly for relatively small values of F. Therefore, the simpler suggestion of Jaeger[3] will be followed, that is, that the value of q_0'' for the cylindrical region be taken equal to the mean of the plane and spherical region values. This leads to the following expression for Q'':

$$Q'' = \frac{2sk\theta_0}{\alpha}\left[\frac{F}{4} + \left(\frac{F}{\pi}\right)^{1/2}\right] = \frac{2sk\theta_0}{\alpha}\left[\frac{yF_m}{4} + \left(\frac{F_m}{\pi}\right)^{1/2} y^{1/2}\right] \qquad (15)$$

This relation is compared with the results of Perry and Berggren in Fig. 2. It is to be noted that Eq. (15) is not accurate for large F and that it results in the wrong value for the asymptotic heat transfer rate.

The variations of Q'' with F for the three geometries are shown in Fig. 2. The effects of curvature in increasing the heat transfer are clearly seen.

The average Nusselt numbers are to be calculated from Eq. (3) with the probability distribution, Eq. (7). For the plane conduction region, Eq. (10) yields

$$\overline{Nu}_n = \frac{2}{\pi^{1/2}}\frac{1}{F_m^{1/2}}\frac{n^{n+1}}{n!}\int_0^\infty y^{n-(1/2)}e^{-ny}\,dy = \frac{C_n}{F_m^{1/2}} \qquad (16a)$$

or

$$\bar{h}_n = C_n\,(\rho ck/\tau_m)^{1/2} \qquad (16b)$$

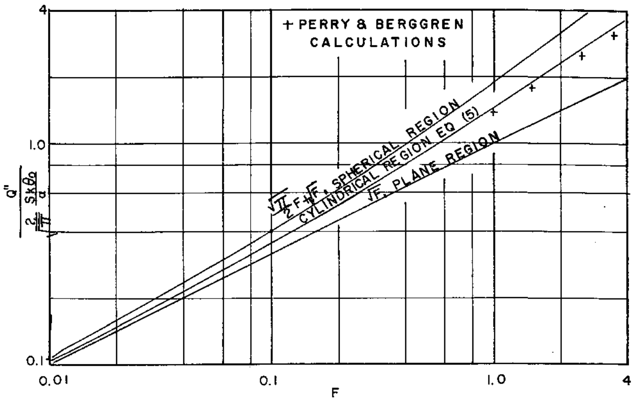

Fig. 2 Heat transfer during a transient

where

$$C_n = \frac{n^{1/2}(2n-1)(2n-3)\ldots3\cdot1}{n!\,2^{n-1}} \qquad (17)$$

Values of C_n for various values of n are given in Table 1.

Table 1 n-dependent constant

n	1	2	3	∞
C_n	1	1.061	1.083	1.128

Equation (16b) indicates that $\bar{h}$ is dependent simply upon the square root of the reciprocal of the most probable time interval. The tabulated values of C_n indicate that the variation of n over its whole conceivable range causes only a 13% change in the heat transfer prediction. The use of the simple exponential probability distribution, Eq. (4b), yields the same form of result, i.e., Eq. (16a). However, the constant C_n is 2.0, and F_m is to be interpreted as the mean value of F.

The result for the spherical region is obtained from Eq. (13):

$$\overline{Nu}_n = \frac{n^{n+1}}{n!}\int_0^\infty\left(\frac{1}{2} + \frac{2}{\pi^{1/2}}\frac{1}{F_m^{1/2}}\frac{1}{y^{1/2}}\right)y^n e^{-ny}dy$$

$$= 1 + \frac{2}{\pi^{1/2}}\frac{1}{F_m^{1/2}}\frac{n^{n+1}}{n!}\int_0^\infty y^{n-(1/2)}\,e^{-ny}dy$$

$$= 1 + \frac{C_n}{F_m^{1/2}} \qquad (18)^{\S}$$

where C_n is again the constant of Eq. (17). That is, the result is merely that for the plane region plus 1.

For the cylindrical region the approximate relation, Eq. (15), is used:

$$\overline{Nu}_n = \frac{n^{n+1}}{n!}\int_0^\infty\left(\frac{1}{2} + \frac{2}{\pi^{1/2}}\frac{1}{F_m^{1/2}}\frac{1}{y^{1/2}}\right)y^n e^{-ny}dy$$

or

$$\overline{Nu}_n = \tfrac{1}{2} + (C_n/F_m^{1/2}) \qquad (19)$$

This is the same as Eq. (18), for the spherical case, except that a $\frac{1}{2}$ appears. The inapplicability of the approximation, Eq. (15), for large F may be seen from Eq. (19). The correct asymptotic value of $\overline{Nu}$ for a long cylinder is zero.

The effects of the random disturbances upon the heat transfer rate may be seen by comparing Eqs. (16a, 18, and 19) with the steady-state results. These latter are the limit as $F_m \rightarrow \infty$. For the plane and cylindrical regions, the limit of $\overline{Nu}$ becomes zero, and for the spherical region $\overline{Nu} \rightarrow 1$, the

§ The importance of random disturbances may be appreciated by comparing the value of $C_n/(F_m)^{1/2}$ with 1.0 (the pure conduction effect). For air at $\frac{1}{3}$ atm, $s = \frac{1}{2}$ ft, and $\tau_m = 30$ sec, the value of $C_n/(F_m)^{1/2}$ is 4.

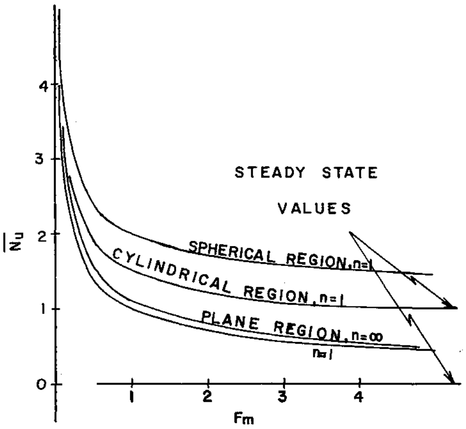

Fig. 3 Effect of disturbances on heat transfer

well-known pure conduction solution. Random convection and pure conduction are compared in Fig. 3, and the differences are seen to be substantial, increasing rapidly with a decreasing time interval between disturbances.

The other transport process to be considered is the filmwise condensation of a saturated vapor at t_∞ on a surface maintained at t_0. It is assumed that there is no liquid motion induced by vapor velocity. The assumption is made that the random disturbances, spaced at time interval τ_c, completely clear the surface of an otherwise stagnant liquid film. A new film forms, and its instantaneous thickness is Y. Assuming that Y remains small compared to the radius of curvature of the solid surface, all geometries may be considered to be a flat surface, shown in Fig. 4. The instantaneous rate of mass addition to the liquid film per unit area is m''.

Employing the original assumption of Nusselt (justified by Sparrow and Gregg[7] for small $c_p(t_0 - t_\infty)/h_{fg}$ for gravity drained films) that the conduction process through the liquid film may be treated as steady-state conduction through a slab, one may write the thermal flux as

$$q'' = (k/Y)(t_0 - t_\infty) = (k/Y)\theta_0 = m''h_{fg} \qquad (20)$$

where h_{fg} is the latent heat of vaporization. The time rate of change of Y is related to m'' as follows:

$$\rho(dY/d\tau) = m'' \qquad (21)$$

Combining Eqs. (20) and (21), one has

$$YdY = (k\theta_0/\rho h_{fg})d\tau$$

This is integrated from $\tau = 0$ to $\tau = \tau_c$, and Q'' is found:

$$Y_c^2 = (2k\theta_0/\rho h_{fg})\tau_c$$

$$Q'' = h_{fg}\rho Y_c = (2\rho k h_{fg}\theta_0\tau_c)^{1/2} = \left(\frac{2\rho s^2 k h_{fg}\theta_0 F_m}{\alpha}\, y\right)^{1/2} \qquad (22)$$

where F_m and y are defined as before. The average Nusselt number is found from Eq. (3) for the probability distribution of Eq. (7):

$$\overline{Nu} = \left(\frac{2h_{fg}}{c_p\theta_0 F_m}\right)^{1/2} \frac{n^{n+1}}{n!} \int^{\infty} y^{n-(1/2)} e^{-ny} dy$$

or

$$\overline{Nu} = \left(\frac{\pi h_{fg}}{c_p\theta_0}\right)^{1/2} \frac{C_n}{F_m^{1/2}} \qquad (23)$$

where C_n is defined in Eq. (17) and is listed in Table 1.

The dependence appearing in Eq. (23) is similar to that in the previous cases presented. However, the dimensionless parameter $\pi h_{fg}/c_p\theta_0$ is very large for many condensation conditions of practical importance. For example, for mercury vapor at 300°C, condensing on a surface at 270°C, the value is 225. In such a circumstance Eq. (23) predicts a high value for the heat transfer rate. Note that the rate in the absence of disturbances approaches zero.

‖ All properties appearing in this equation are for the liquid phase.

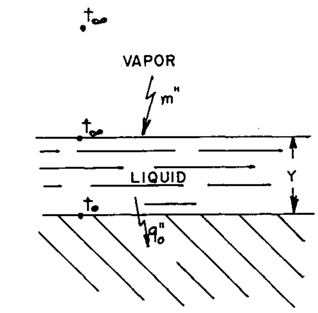

Fig. 4 Filmwise condensation

Conclusion

The results for the cases analyzed indicate that random impulses would have a large effect on transport rates in circumstances where forced fluid motion is not assured. Although only simple heat conduction and vapor condensation processes were treated specifically, the same considerations would apply for many other types of transport processes. In particular, the conduction solutions presented apply to mass diffusion processes that might arise, for example, in transpiration cooling or in metabolic respiration. If the mole fraction of the diffusing chemical species is small, the same solutions are applicable if the thermal diffusivity in the Fourier number is replaced by the chemical diffusivity.

The effects of disturbances are shown to be very large in a condensation process. Boiling might be analyzed similarly. The result of condensation suggests that it might be more desirable intentionally to introduce closely spaced disturbances into a condenser rather than to arrange for the drainage of extensive condensate films by, for example, controlling the motion of the vapor. The arrangement of surface undoubtedly would be different in that case.

The principal assumptions that support the point of view and methods of analysis of this paper are that there are continuing disturbances that are appreciable compared to any steady "body" forces present and that these disturbances cause relative displacements that are the order of size of the various transfer surfaces of interest. Detailed consideration of a proposed design would permit the assessment of these assumptions for that design. Perhaps in the future, measurements can be made on operating space devices which will permit a general characterization of the basic parameters in such processes. Such measurements also will suggest the proper probability distribution for disturbances of this type.

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